The effect of fibre orientation on the interfacial shear stress in short fibre-reinforced polypropylene

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Two methods for the determination of interfacial shear stress and the orientation effects in well aligned short fibre-reinforced polypropylene specimens subjected to uniaxial tension have been discussed. Both methods are based on the stress-transfer model due to Kelly and Tyson, but the orientation effects are taken into account by different procedures. In the first procedure the stress and strain along the load axis are considered and the orientation factor is found to be close to $\cos^4\theta$ for $\theta \leqslant 45°$. In the second procedure the fibre orientation is taken into account by resolving stress and strain in the fibre axis direction. The transverse strain is also considered with the help of experimentally determined values of Poisson's ratio. The second procedure was found to be very satisfactory and it also confirms the assumption of linear dependence of interfacial stress on the tensile stress in the fibre direction, as postulated in an earlier study, for all orientations.

1. Introduction

The mechanical properties of a short fibre-reinforced polymer composite depend on several factors like fibre length, fibre orientation and the nature of the bond between the fibres and the matrix. The roles of these factors have been clearly demonstrated in a recent study [1] on a graphite fibre–epoxy resin system in which fibre length, fibre orientation and the nature of the interface were varied independently and the effects on composite properties were determined. The influence of fibre length and fibre orientation on the strength and stiffness of a glass–epoxy system have also been reported [2].

The fibre orientation effect for an aligned fibre composite is sometimes represented by an orientation factor, which was taken to be $\cos^4\theta$ by Cox [3], where θ is the inclination of the fibres to the load axis. However, a more rational explanation of orientation effects can be obtained using macromechanical analysis [4] of strength and stiffness properties [1, 2, 5]. On the other hand, the effect of fibre length (or equivalently the fibre aspect ratio) can be considered theoretically using one of the several models of stress transfer from matrix to fibre. These models differ greatly in sophistication and applicability but one due to Kelly and Tyson [6] is very simple and effective for short fibre-reinforced thermoplastics [5, 7–9]. This model considers the stress distribution due to a single short fibre embedded in a matrix when the system is subjected to a uniaxial load in the fibre axis direction. In applying this model, Bowyer and Bader [7] assumed that the interfacial shear stress (τ) is independent of the composite strain, and that the effect of fibre inclination to the load axis can be accounted for by a scalar factor, i.e. the fibre orientation factor (η). These authors suggest a numerical algorithm to determine both τ and η from two points on the stress–strain curve obtained in a uniaxial tension test on the composite specimen. On the other hand, Mittal and Gupta [5] restricted the application of the Kelly–Tyson model to specimens in which fibres are nominally aligned to the load axis, as is intended in the model. Also, it was argued that the interface cannot be assumed to be subjected to the same shear stress for all composite strains. Rather τ increases gradually till it reaches the maximum value, i.e. interfacial bond strength, when the composite fails by fibre pull-out. Thus it was assumed that $\tau = K\sigma_c$, where σ_c is the applied composite stress and K is a constant which is a characteristic of the composite system. It was also suggested that K can depend on the fibre inclination θ.

The aim of the present work is to ascertain if the variation of K with θ can be obtained within the framework of the Kelly–Tyson model. Alternatively, we also seek a rational way to adapt the Kelly–Tyson model to an inclined fibre system, thus eliminating the need for K to vary with θ. In that case, K will be a representative interface parameter.

2. Basic equations

In the Kelly–Tyson model, an important concept of critical fibre length (l_c) is used and it is given by

$$l_c = \frac{E_f r_f \varepsilon_c}{\tau}, \qquad (1)$$

where E_f and r_f are, respectively, the fibre modulus and fibre radius, ε_c is the composite strain in the direction of the fibre axis and τ is the interfacial shear stress. If the composite consists of a large number of fibres of different lengths but aligned parallel to the load direction, then the composite stress is obtained by

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applying the well-known rule of mixtures in which the fibre contribution for the subcritical fibres and supercritical fibres is combined with the contribution of the matrix. In other words,

$$\sigma_c = X + Y + Z$$

where

$$X = \sum_{\text{subcritical}} \tau L_i V_i / 2r_f \quad (2)$$

$$Y = \sum_{\text{supercritical}} E_f \varepsilon_c \left(1 - \frac{E_f \varepsilon_c r_f}{2L_j \tau}\right) V_j, \quad (3)$$

$$Z = V_m (\sigma_m)_{\varepsilon=\varepsilon_c} \quad (4)$$

In Equation 2, the summation is carried out for all fibres shorter than the critical length while in Equation 3, fibres longer than the critical length are considered. V_i and V_j are, respectively, the volume subfractions of fibres of length L_i and L_j. These volume subfractions are obtained by plotting a histogram of fibre lengths. In Equation 2, $\tau L_i/2r_f$ represents the average fibre stress for a fibre of length L_i. Similar remarks hold for the summand in Equation 3. V_m is the volume fraction of the matrix and $(\sigma_m)_{\varepsilon=\varepsilon_c}$ is the matrix stress corresponding to strain ε_c. As mentioned earlier, Bowyer and Bader [7] introduced a scalar factor η in order to determine the composite stress for a specimen in which all the fibres are inclined at an angle θ to the load axis. The modified expression is

$$\sigma_c = \eta(X + Y) + Z \quad (5)$$

whereas X and Y are as in Equations 2 and 3. Z was simplified as

$$Z = E_m \varepsilon_c V_m$$

In other words, the matrix was considered as an elastic medium of modulus E_m. The inadequacy of a single scalar factor in Equation 5 to represent the fibre orientation effect was discussed in a previous article [5].

Instead, the effect of fibre orientation can be considered in a rational way with reference to Fig. 1, where the applied stress σ_c in the loading direction has been resolved into three stress components $\sigma_c \cos^2\theta$, $\sigma_c \sin^2\theta$ and $\sigma_c \sin\theta\cos\theta$. In particular, the stress component in the fibre direction is

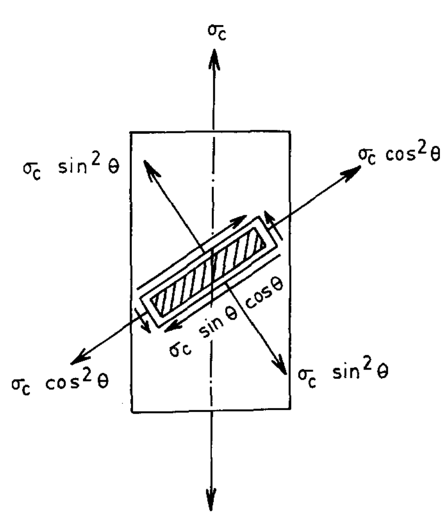

Figure 1 Resolved components of stress in inclined fibre case.

$$\sigma_{\theta c} = \sigma_c \cos^2\theta \quad (6)$$

The evaluation of the strain component in the fibre direction is more complex. The Poisson's ratio of a fibre-reinforced composite, ν, is a function of the inclination of fibres to the load axis. For elastic deformation, $\nu(\theta)$ is independent of ε_c, but for large deformation, $\nu(\theta)$ depends on ε_c. It is known that for off-axis tensile tests, i.e. when fibres are inclined to the load axis, the specimen undergoes shear deformation [4], which is prevented by the grips of the testing machine. Consequently, the deformation becomes non-homogeneous near the grips. As an approximation in the analysis presented here, this non-homogeneity is neglected and the component of strain $\varepsilon_{\theta c}$ along the fibre axis is given by the following relation:

$$\varepsilon_{\theta c} = \varepsilon_c(\cos^2\theta - \nu(\theta)\sin^2\theta) \quad (7)$$

When the stress and strain components in the fibre direction, i.e. $\sigma_{\theta c}$ and $\varepsilon_{\theta c}$, are considered, the Kelly–Tyson model becomes applicable and consequently Equations 2 to 4 can be used.

3. Experimental details

3.1. Sample preparation

Profax PC 073, a commercial glass fibre-filled polypropylene containing 30% by weight of short fibres and supplied by Hercules Inc., Belgium, in granular form, was used as the starting material. It was extruded to get sheet of 150 mm width and 0.75 mm thickness. The short fibres were quite well aligned along the extrusion direction. A sheet of unreinforced polypropylene was also extruded under similar conditions.

3.2. Measurement of fibre-length distributions

Small chips of the composite material were burnt at 450° C for $4\frac{1}{2}$ h in a muffle furnace. In order to obtain a proper statistical distribution of fibre lengths, the lengths of 400, 800, and 1200 fibres obtained from the composite sample were measured, using an image analyser (Model 30 MOP, Kontron GMBH, W. Germany). It was found that a sample size of 800 fibres gives an appropriate distribution.

3.3. Stress–strain behaviour

Dumb-bell shaped specimens with various fibre orientations to the direction of extrusion were cut from the extruded sheet. The nominal width of the specimens was 5 mm and the length of the parallel portion was 55 mm. These specimens were tested on an Instron tensile tester at a strain rate of $1\%\ \text{min}^{-1}$. The longitudinal strain was measured on a central region of 20 mm length in order to keep the error due to non-homogeneous deformation (as mentioned in the previous section), which occurs near the grips, to the minimum. The ambient temperature during testing was 20 ± 2° C. Samples were tested to obtain stress–strain curves up to a strain of 2%. This limitation was due to the limited linear range of the clip gauge as discussed below.

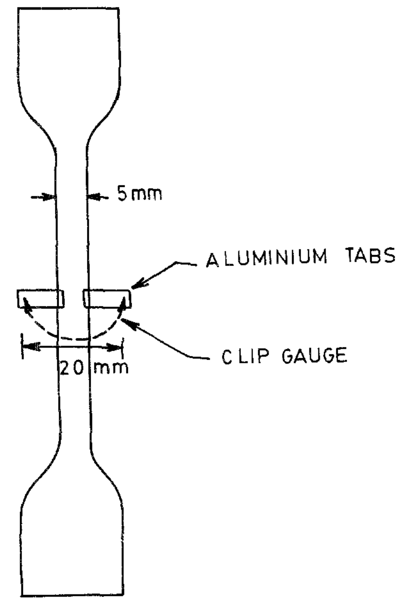

Figure 2 Clip gauge arrangement for measuring Poisson's ratio.

3.4. Measurement of Poisson's ratio

As stated above, the values of Poisson's ratio at different longitudinal strains are also needed for these samples. The measurement of Poisson's ratio on thin and narrow specimens presents many difficulties because any suitable strain gauge must have a very small gauge length and applicability to large strains (up to about 2%). Also, it should not add to the specimen stiffness. Resistance strain gauges satisfy these requirements but a direct installation of strain gauges in the transverse direction showed excessive drift at higher strains, which could be eliminated. In view of these difficulties, a clip gauge of phosphor bronze was fabricated, on which two wire resistance strain gauges were mounted. The gauge length of the clip gauge was 20 mm. For the measurement of transverse strain, the gauge is held across the two metal tabs which are adhesively bonded to the specimen as shown in Fig. 2. The contact between the clip gauge and the metal tabs was maintained using either rubber bands or a thin wire looped around the tabs. This arrangement was found to be stable and reproducible in the range of 0.25 to 0.75% transverse strain. Before using the clip gauge it was calibrated on a special device using a screw micrometer. The output of the strain gauges was linearly proportional to the opening displacement of the gauge.

TABLE I Ultimate tensile strength of reinforced and unreinforced polypropylene

Sample	Ultimate tensile strength (MPa)
Polypropylene	19.62
Reinforced polypropylene	
0°	67.6
15°	52.3
25°	50.4
35°	40.9
45°	38.6

4. Experimental results and analysis

4.1. Experimental results

The fibre length distribution data are shown in Fig. 3. The average fibre length and standard deviation were found to be 0.4 and 0.2 mm, respectively, while fibre radius was 0.0075 mm. The stress–strain curves for composite samples having various fibre orientations to the load axis and for the base polymer are shown in Fig. 4. As is obvious from Fig. 4, the maximum reinforcement effect is obtained in case of fibres aligned in the direction of applied load. The ultimate tensile strengths for the composite samples are shown in Table I. The experimental values of Poisson's ratio as a function of composite strain are plotted in Fig. 5 for composites with different fibre orientations.

4.2. Analysis of results

The experimental results have been analysed with the aim of comparing various approaches for the determination of interfacial shear stress and the influence of fibre orientation. Three procedures of analysis have

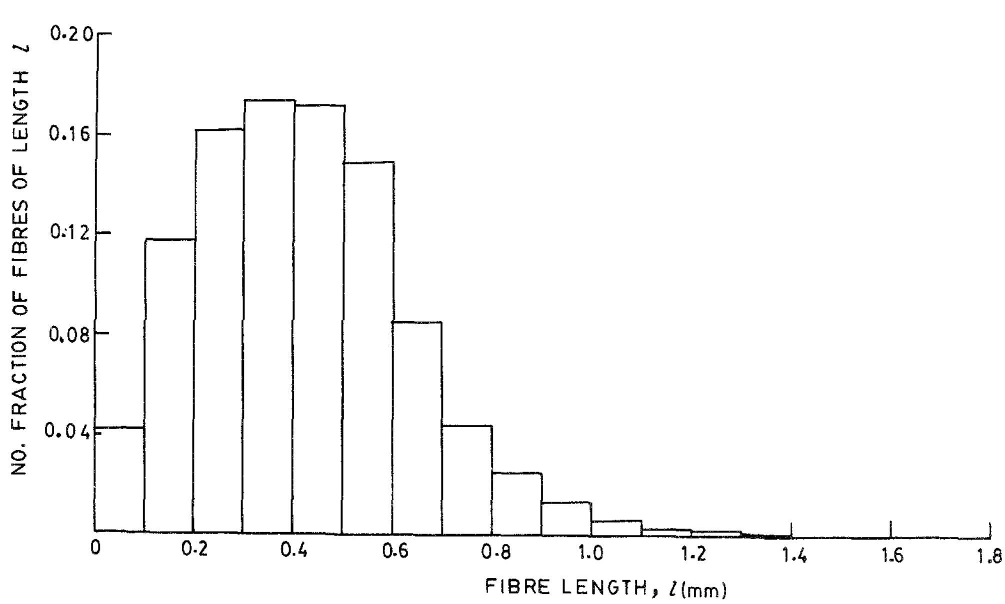

Figure 3 Histogram of fibre lengths.

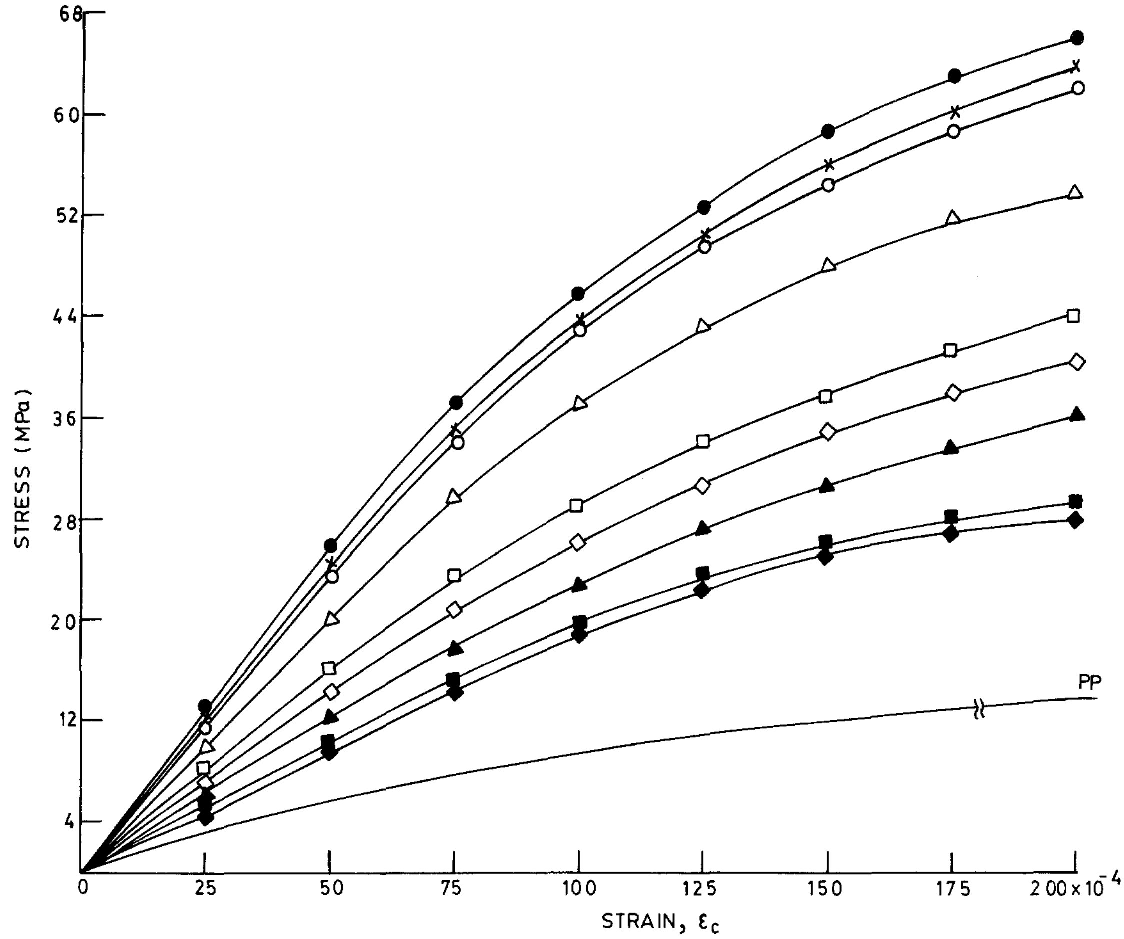

Figure 4 Stress–strain curves for the unreinforced sample (PP) and reinforced samples: θ = (◆) 90°, (■) 60°, (▲) 45°, (◇) 35°, (□) 25°, (△) 15°, (○) 10°, (x) 5°, (●) 0°.

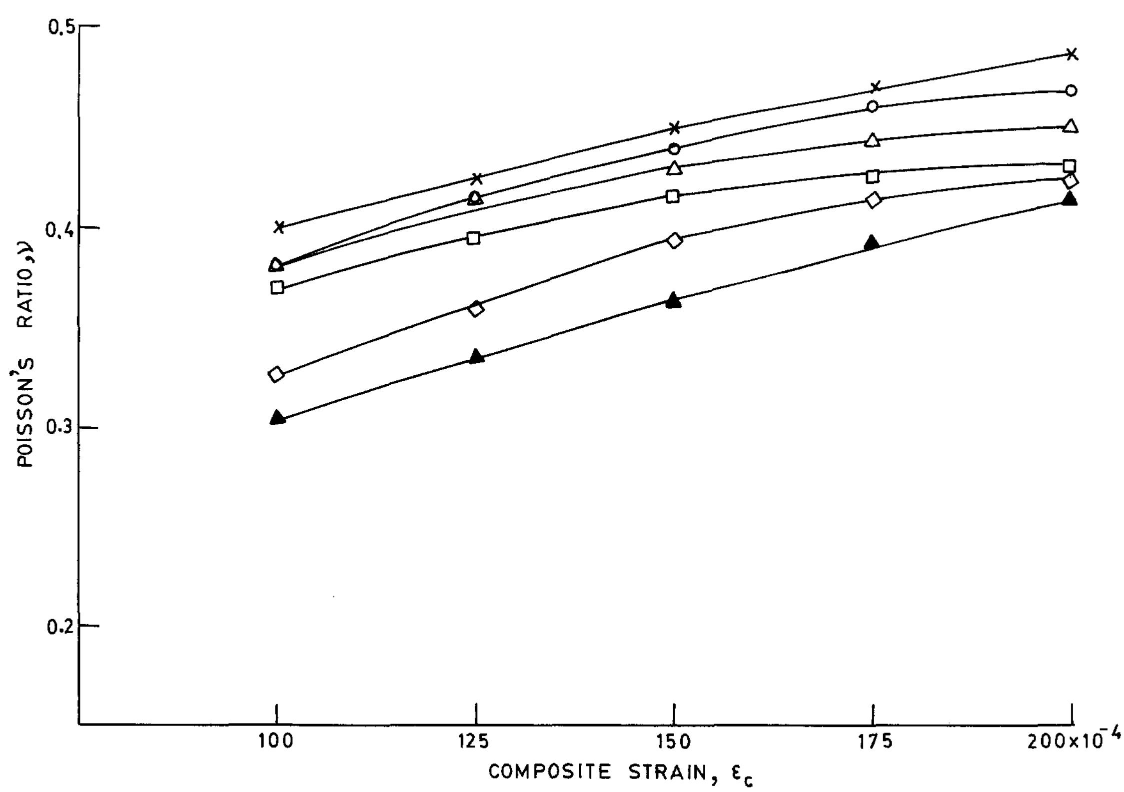

Figure 5 Poisson's ratio as a function of composite strain: θ = (▲) 45°, (◇) 35°, (□) 25°, (△) 15°, (○) 10°, (x) 5°.

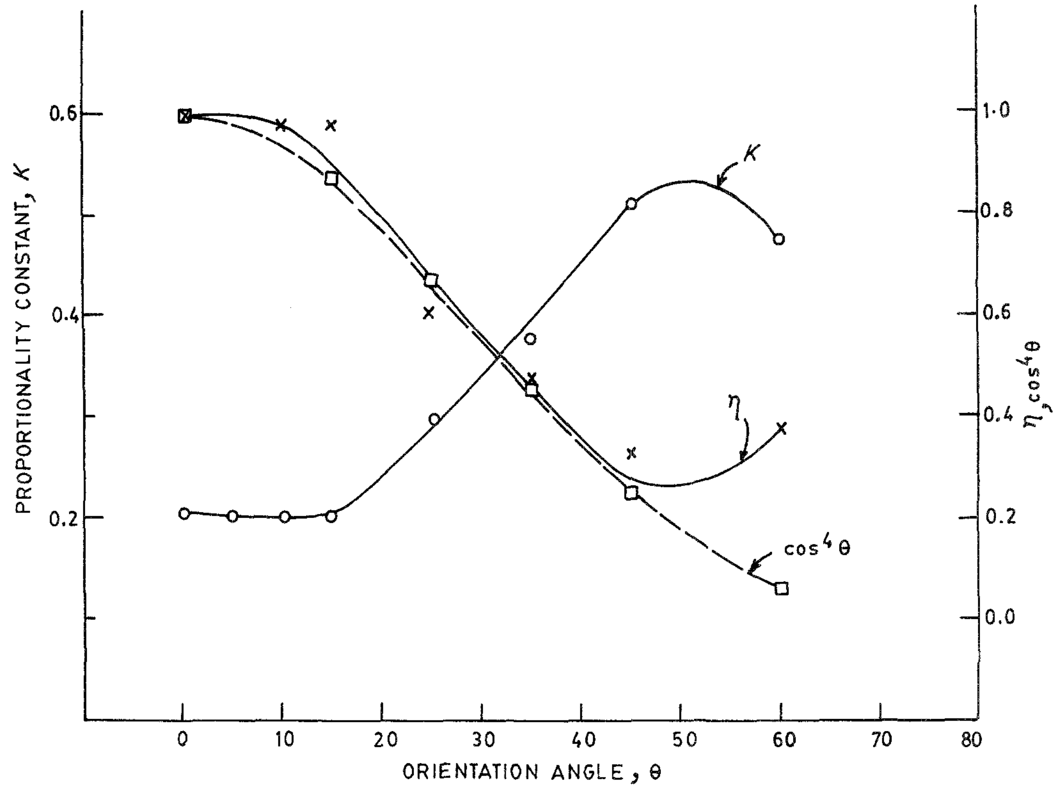

Figure 6 K, η and $\cos^4\theta$ as functions of fibre orientation for Procedure (ii).

been considered in this paper:

(i) assuming that τ is constant and the orientation effect is given by a single factor η;

(ii) assuming that $\tau = K\sigma_c$, and the orientation effect is given by η as well as the dependence of K on orientation; and

(iii) assuming that $\tau = K\sigma_{\theta c}$ and considering the effect of transverse strain while calculating the critical fibre length. Again, K can depend on θ.

4.2.1. Procedure (*i*)

This is essentially that due to Bowyer and Bader [7], but whereas Bowyer and Bader considered only two points on the stress–strain curve ($\varepsilon_c = 1\%$ and $\varepsilon_c = 2\%$), we have considered several points in this range so as to obtain a more representative value of η. In this procedure several values of τ are tried and substituted in Equations 2 to 5 to determine η. The selection criterion for τ is that η remains unchanged or varies within a small range only for all strain values in the range considered. When this procedure was applied to stress–strain curves corresponding to various values of θ, it was found that a suitable τ could not be obtained for any orientation state. For all orientations and for all choices of τ, the variation of η was large.

4.2.2. Procedure (*ii*)

In this procedure various K values are considered and $\tau = K\sigma_c$ obtained. The remaining procedure is the same as in (i). Above the selected value of K is one which gives a constant η or one showing only a small variation in the range of strains considered. In the previous paper [5] this analysis has already been done, for $\theta = 0°$. The analysis has been extended in this paper to other values of θ. On repeating the analysis for $\theta = 0°$, it was noticed that in the previous paper [5] there was a slight error in summing up for the fibre length distributions. This has now been corrected. It results in some changes in the actual value of critical length and also in the contribution of the subcritical and supercritical fibres, but the basic arguments proposed in the previous paper are not affected. The angles for which the present analysis was done were $\theta = 0, 5, 10, 15, 25, 35, 45$ and $60°$. The results of this procedure, i.e. K, η and $\cos^4\theta$, are shown as functions of θ in Fig. 6. This model showed convergence for all values of θ that were considered, i.e. up to $\theta = 60°$. From Fig. 6 it is seen that the value of η is close to $\cos^4\theta$ and K is monotonically rising up to $\theta = 45°$. However, when $\theta > 45°$, there is an appreciable difference between η and $\cos^4\theta$. Also K shows a reversal of trend. These factors indicate that Procedure (ii) is not very satisfactory beyond $\theta = 45°$.

Comparing the usefulness of Procedures (i) and (ii) it is seen that the assumption $\tau = K\sigma_c$ is more acceptable than $\tau =$ constant for all composite strains considered. However, the apparent weakness of Procedure (ii) for $\theta > 45°$ led us to re-examine the application of the Kelly–Tyson model in the case of an inclined fibre so that the orientation effects could be appropriately taken into account. Consequently Procedure (iii) was evolved.

4.2.3. Procedure (*iii*)

Here the orientation effect is accounted for by considering the relevant stress and strain components, as given by Equations 6 and 7, in the fibre direction. Thus the problem essentially reduces to one in which fibres are parallel to the load axis ($\eta \approx 1$). The effect of other components of stress, shown in Fig. 1, can possibly influence the value of K and hence K can depend on θ. The numerical procedure is otherwise identical to Procedure (ii). Different values of K were again tried

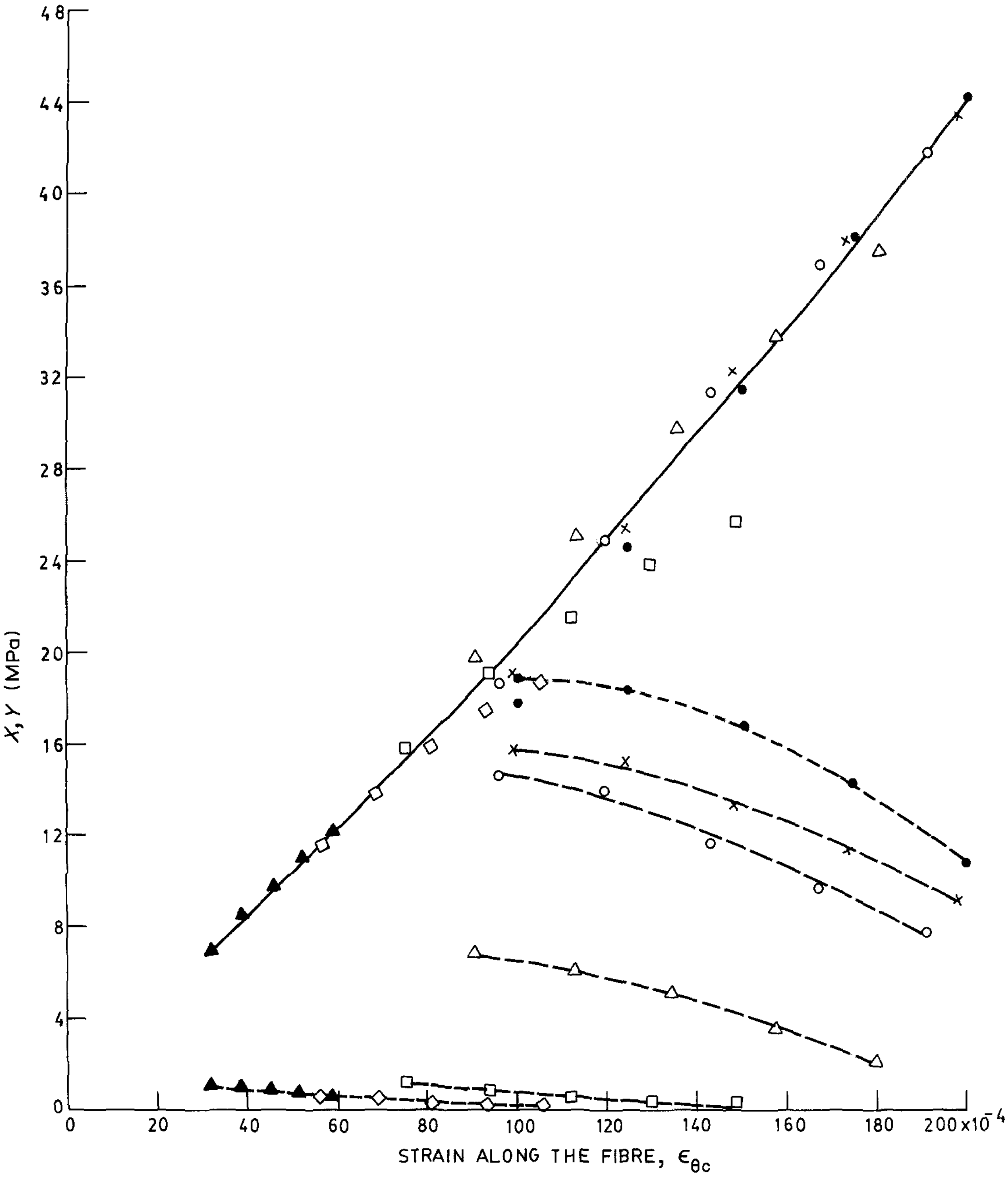

Figure 7 The parameters (——) X and (- - -) Y as functions of composite strain obtained from Procedure (iii): θ = (▲) 45°, (◇) 35°, (□) 25°, (△) 15°, (○) 10°, (x) 5°, (●) 0°.

and the one which gives η closest to 1.0 for all strains was adopted. Using this procedure, very encouraging results were obtained for all fibre orientations for which $v(\theta)$ was available. The acceptable values of K for various orientation are shown in Table II. Interestingly, K is varying in a small range between 0.173 to 0.21, thus indicating that the influence of other components of stress is only marginal.

TABLE II K values calculated by Procedure (iii) for various orientation angles

Orientation angle (deg)	K value by Procedure (iii)
0	0.21
5	0.206
10	0.206
15	0.195
25	0.179
35	0.173
45	0.176

The results of this procedure are illustrated with the help of Figs 7 and 8. Fig. 7 shows the contributions X and Y of the subcritical and supercritical fibres, respectively, to the total composite stress in the fibre direction as functions of the strain along the fibres. The contribution of supercritical fibres is little at large strains, and it reduces even more for the specimens with a higher degree of fibre orientation, thus showing that in composite specimens made from the commercially available material, most of the fibres are subcritical and hence the efficiency of reinforcement is poor. In Fig. 8 the variation of τ with the applied strain is shown. It is observed that the more the fibre inclination to the load axis, the less is the interfacial shear and hence the less is the load transfer to the fibres. In the extreme case when θ = 90°, no load transfer is expected.

4.3. Physical significance of the factor K

The analysis presented in this paper supports the

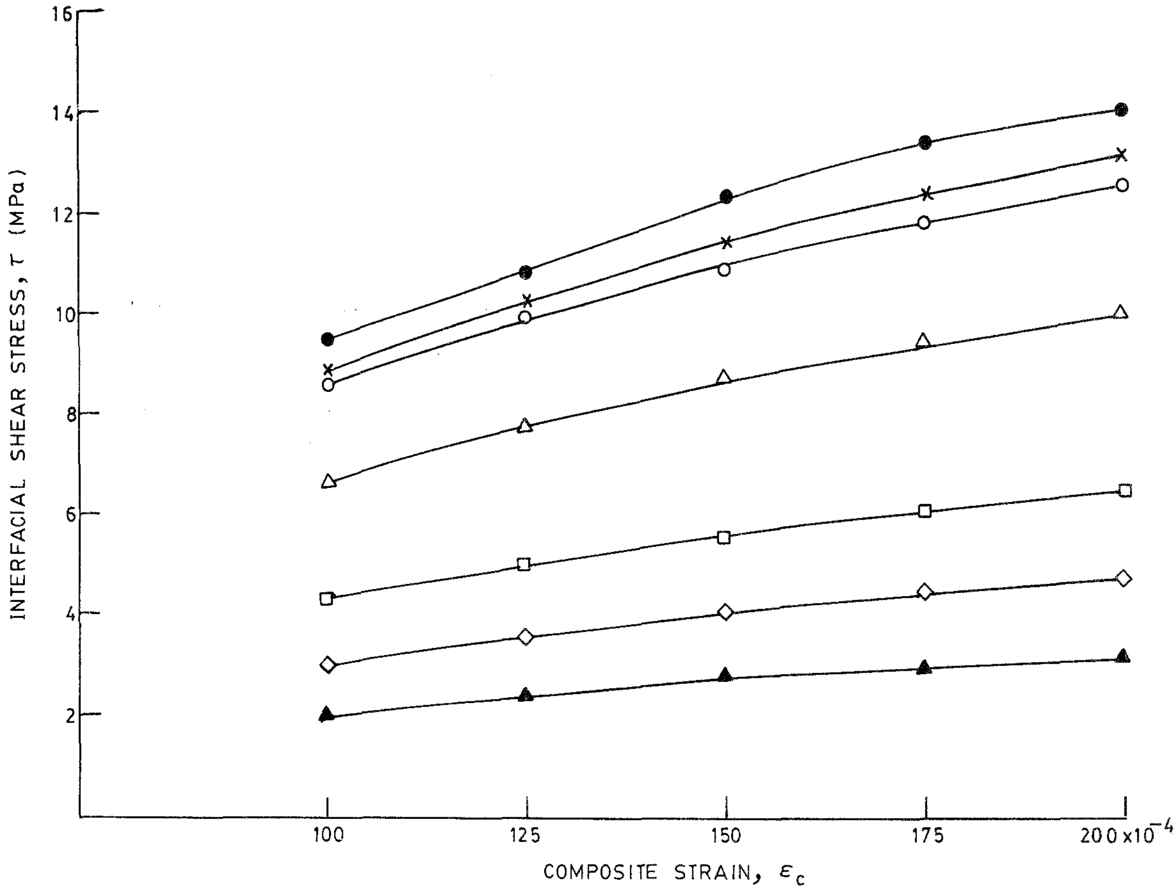

Figure 8 Interfacial shear stress as a function of composite strain: θ = (▲) 45°, (◇) 35°, (□) 25°, (△) 15°, (○) 10°, (x) 5°, (●) 0°.

validity of the equation for τ proposed in the earlier paper [5], namely $\tau = K\sigma_c$ for $\theta = 0°$. When applied to the off-axis cases, it became clear that the actual stresses on the fibre, the matrix and the interface needed to be considered rather than the applied stresses. This led us to the use of the equation $\tau = K\sigma_c \cos^2\theta$ and its use gave a value of K around 0.19 ± 0.02. Thus K is a constant for the system considered and, as suggested in the previous paper [5], it may be analogous to a frictional coefficient. The value of K is characteristic of the nature of bonding between fibres and the matrix just as the coefficient of friction depends on the type of contact between two surfaces.

5. Conclusions

A procedure has been established to consider the orientation of fibres to the load axis in a rational way in order to determine the interfacial shear stress. This procedure is an adaptation of Kelly–Tyson model to off-axis loading. The alternate method of using the unmodified Kelly–Tyson model and deducing the orientation effects as a factor η has only a limited usefulness.

The present study also confirms the assumption postulated in previous article [5], that the interfacial shear stress varies linearly with the tensile stress on the specimen. This assumption is more consistent than the one due to Bowyer and Bader [7], namely that the shear stress is constant for all strains. The load transfer from matrix to the fibres in short-fibre reinforced thermoplastics is characterized by a single parameter which varies little with fibre inclination to the load axis. If the distribution of fibre orientations is known for a given sample, then the composite stress–strain behaviour can be predicted using these results.

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